DETERMINISABILITY & DETERMINISTIC SEPARABILITY & TIMED AUTOMATA

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Summary

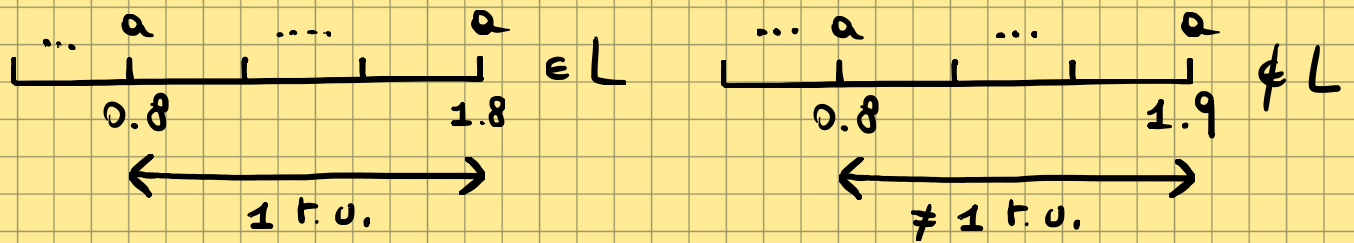
TITLE : Deterministic membership and separability problems for Timed automata.

- DTA are less expressive than NTA : last letter appeared 1 t.u. before. (Note it is unambiguous).
  Intuition : need ∞-many clocks to recognise L deterministically.
  L is 1-NTA recognisable : guess the second-last appearance of the last letter reset the clock and measure 1 t.u.
- Deterministic membership is undecidable for $K \geq 2$ clocks. Decidable for $K=1$ clocks (w20 techniques)
- Future directions : UTA membership & separability problems.
- Open problem : DTA separability (however games are undecidable).

- Open problem : is there a NTA & coNTA language which is not DTA?
  Not true for 1-NTA : 1-NTA & 1-coNTA $\Rightarrow$ DTA.

DETERMINISTIC TA LESS EXPRESSIVE

L = "last letter occurred 1 t.u. earlier", over $\Sigma = \{a\}$



- NTA for L with 1-clock:
guess the second-last occurrence and measure 1 t.u.

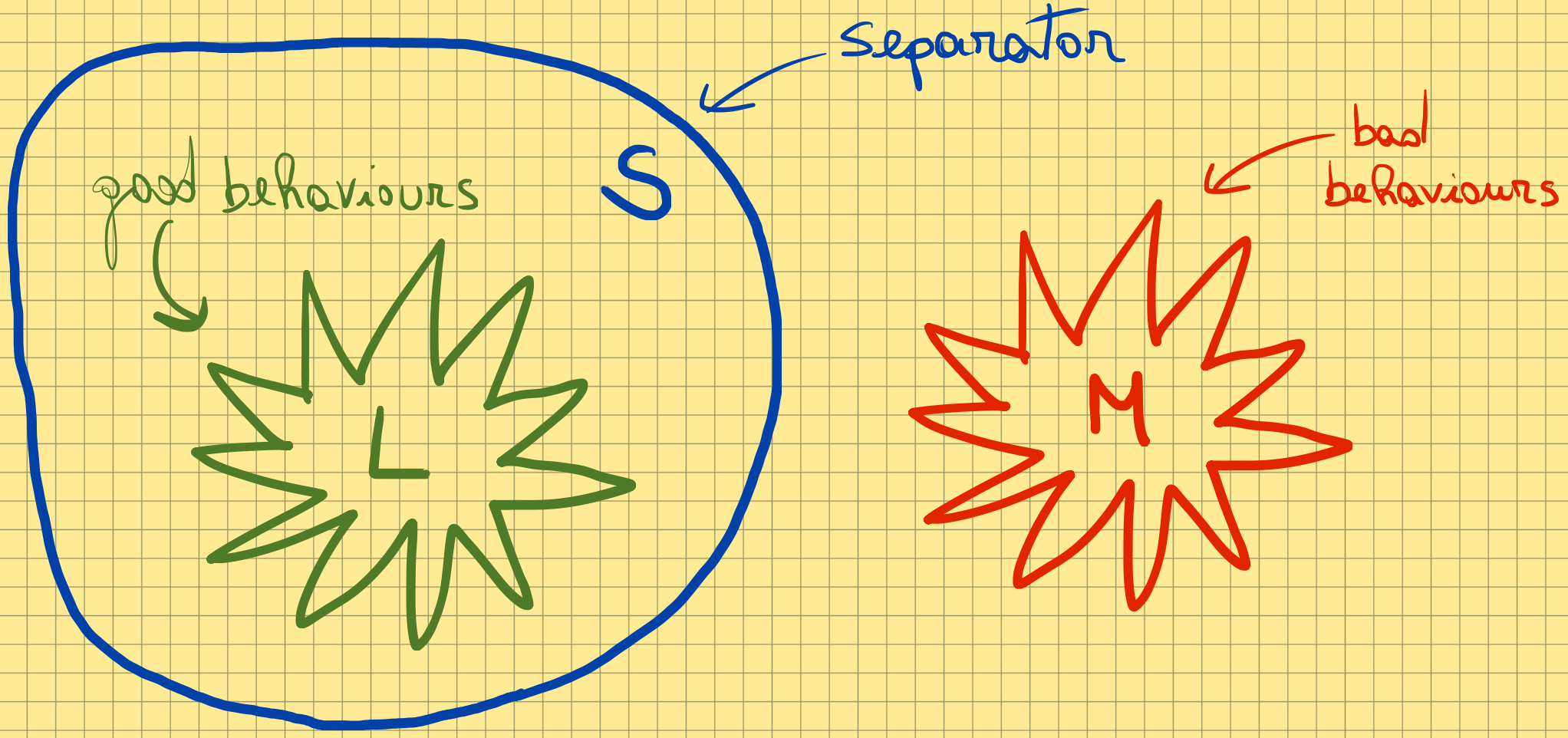
- No DTA for L: deterministically ∞-many clocks are required to store unboundedly many timestamps.
→ any DTA with k clocks can be "fooled" since some important timestamp must be forgotten.

DETERMINISABILITY

INPUT \ OUTPUT	DTA k clocks constants $\leq M$	DTA k clocks	DTA
NTA ≥ 2 clocks	UNDECIDABLE (1)	UNDECIDABLE (1)	UNDECIDABLE (1)
NTA 1 clock ε-transitions	UNDECIDABLE (1)	UNDECIDABLE (1)	UNDECIDABLE (1)

(1) [FinKel '06], [Tripakis'06]

DETERMINISTIC SEPARABILITY



L, M are **complex** : NTA. S is **simple** : DTA.

WHY SEPARABILITY?

- Simple explanation of disjointness. → checkable!
  → Disjointness undecidable, but separability decidable.
  Example: CFL and piecewise-testable separability.

[ ]

- Generalisation of learning from positive & negative examples.
- Approximate determinisability with guarantees.
  → Determinisability undecidable, but det. sep. decidable.
  Example: this work!
- Deeper understanding of nondeterminism vs. determinism.

RECENT WORKS on SEPARABILITY

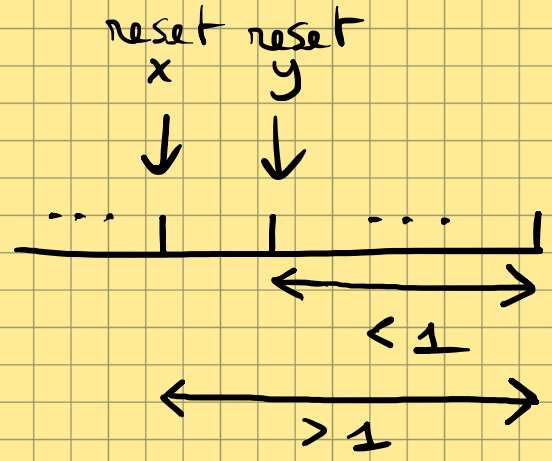
DETERMINISTIC SEPARABILITY NONTRIVIAL

L = "last letter occurred 1 t.u. earlier" (1-clock NTA).

M = complement of L (2-clock NTA).

$w \in M$ iff either: 1) $|w| = 1$, or

2) w elapses < 1 time unit, or



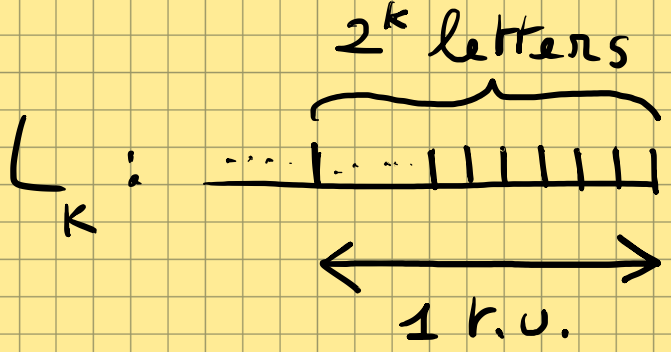
(2 clocks)

→ L, M not DTA separable.

→ 2 clocks necessary: 1-NTA ∩ 1-coNTA = DTA (unpublished).

→ Stronger conjecture: disjoint 1-NTAs always DTA-separable.

SEPARATOR NEEDS MANY CLOCKS



NTA with $2K+2$ clocks :

1 clock for measuring 1 t.u. (as before).

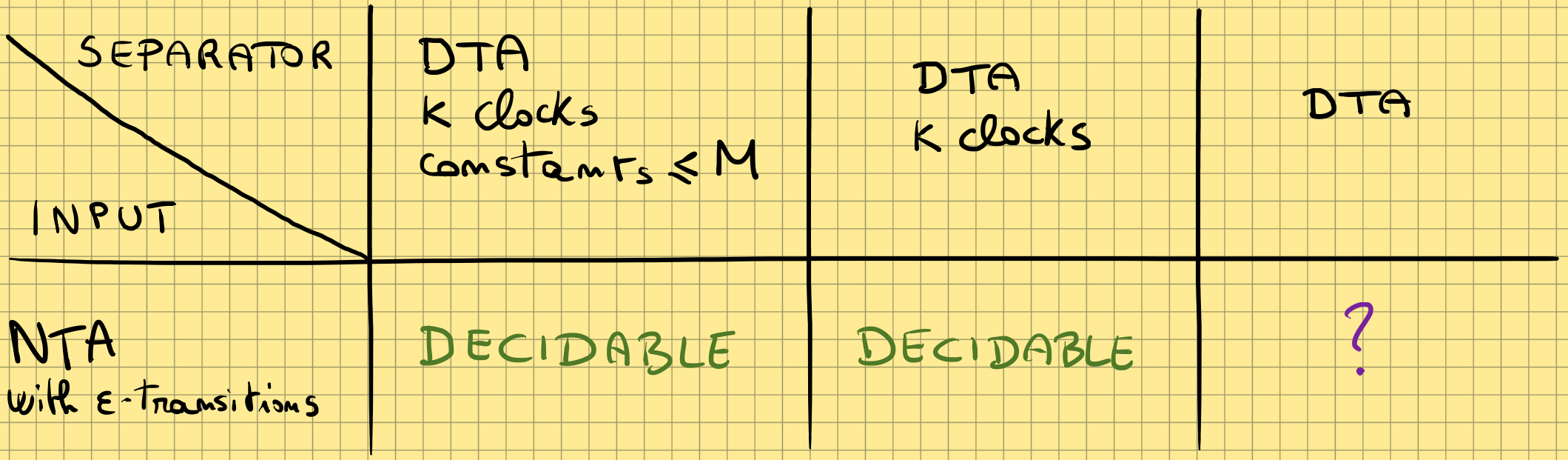
1 clock for strict monotonicity.

K clocks for counting 2^K letters in binary.

M_K = complement of L_K, also NTA with $2K+2$ clocks.

A DTA for L_K needs 2^K clocks.

RESULTS ON DETERMINISTIC SEPARABILITY



RESULTS ON DETERMINISTIC SEPARABILITY

SEPARATOR / INPUT	DTA k clocks constants $\leq M$	DTA k clocks	DTA
NTA with ε-transitions	DECIDABLE	DECIDABLE	?
	REDUCTION ↓	REDUCTION ↓	REDUCTION ↓
Synthesis of a finite-memory winning strategy with ...	DECIDABLE	DECIDABLE	UNDECIDABLE
	k clocks constants $\leq M$	k clocks	arbitrary

SOLVING DETERMINISTIC SEPARABILITY

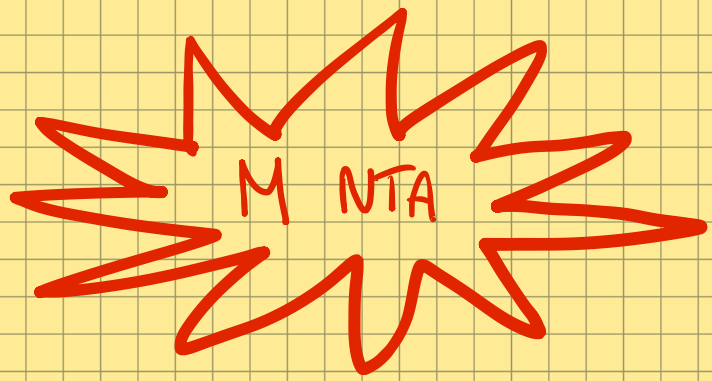
IDEA :

DETERMINISTIC SEPARABILITY $\xrightarrow{\text{reduces to}}$ TIMED GAMES

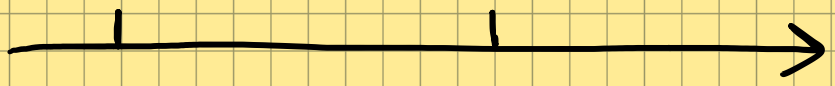
CORRECTNESS:

DETERMINISTIC SEPARATORS with k clocks, constants $\leq M$ $\xleftrightarrow{\text{bijection}}$ FINITE-MEMORY WINNING STRATEGIES k clocks, constants $\leq M$

DETERMINISTIC SEPARABILITY GAME



INPUT : $(a, 0.2)$ $(b, 1)$ ... $= w$ timed actions



SEPARATOR : reject accept ... instantaneous untimed response

Winning condition for INPUT :

1) prefix of $w \in L$ & SEPARATOR rejects, OR
2) prefix of $w \in M$ & SEPARATOR accepts.

NTA language over $\{a, b\} \times \{\text{accept}, \text{reject}\}$

NOVELTY: CONSTANT SYNTHESIS

TIMED BÜCHI-LANDWEBER GAMES

Winning condition			
β_2 INPUT	DECIDABLE	DECIDABLE	?
β_2 OUTPUT			

DETERMINISABILITY

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NTA 1 clock no ε-tr.	DECIDABLE	DECIDABLE	UNDECIDABLE

CONCLUSIONS

- Similar results for register automata over tractable data domains such as $(\mathbb{A}, =)$, $(\mathbb{Q}, \leq)$, ...

- Open problem: given L, M NTA s.t. $M = L^c$.
  decide whether L is DTA? (promise problem)