Formal verification - Homework 01

Deadline: 2026/04/22

LTL

Consider LTL extended with a new binary operator $\oplus$,

$$\varphi, \psi ::= p \mid \neg\varphi \mid \varphi \wedge \psi \mid \mathbf{X}\varphi \mid \varphi\mathbf{U}\psi \mid \varphi \oplus \psi.$$

The semantics of the new operator is defined as follows. For an infinite word $w = a_0 a_1 \cdots$ over 2^P, write $w_{\text{even}} = a_0 a_2 \cdots$ and $w_{\text{odd}} = a_1 a_3 \cdots$. Then,

$$w \models \varphi \oplus \psi \iff w_{\text{even}} \models \varphi \text{ and } w_{\text{odd}} \models \psi.$$

Exercise 1.

1. *Can we express the $\oplus$ operator in standard LTL?*
2. *Present an algorithm converting an LTL formula with $\oplus$ into an equivalent alternating Büchi automaton.*

Pushdown systems

Let P be a pushdown system. Recall that transition rules are of the form

$$p, \gamma \to q, w$$

where p, q are control locations, γ is the stack symbol that is popped off the stack, and w is a word of stack symbols that is pushed onto the stack. Recall that a configuration is a pair $c = (p, w)$ where p is a control state and w is the stack content. The *forward reachability set* of a set of configurations S is the set of configurations that can be reached from some configuration in S by a finite sequence of transitions.

Exercise 2. *Let S be a regular set of configurations. Is the forward reachability set of S regular? Can one compute a finite automaton recognising it?*

Well quasi-orders

Let Σ be a finite alphabet. Recall that two languages $L, M \subseteq \Sigma^*$ are *separable* by a language $S \subseteq \Sigma^*$ if $L \subseteq S$ and $M \cap S = \varnothing$.

Exercise 3. *Consider the (scattered) subword ordering $\sqsubseteq$ on Σ^*. Show that any two disjoint downward-closed languages $L, M \subseteq \Sigma^*$ are separable by a regular language.*

Lossy channel systems

Exercise 4. *Show that the forward reachability set for lossy channel systems is regular. Can one compute a finite automaton recognising it?*